Global optimisation in PCA shape space using level sets

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Purpose

Statistical shape models have been widely used for image segmentation and shape estimation from sparse sets of landmarks [1,2]. Existing works on optimisation in shape space aim at finding a single instance from the statistical shape model that best approximates the input data, subject to some regularisation constraint. In certain cases, it may be interesting to find all instances of the shape model that meet a certain criterion. For example, one may be interested in estimating which range of population falls within a given anatomical criterion, thus establishing a partition of the shape space into "valid" and "invalid" shapes. In this work, we propose a method for global optimisation of shape constraints that effectively finds *all* instances in the PCA (principal component analysis) shape space that meet a certain criterion.

Methods

The method is based on level sets in the parametric shape space defined by PCA. PCA is a multivariate factor analysis technique aiming at finding a low-dimensional manifold in the space of the data, such that the distance between the data and its projection on the manifold is small [3]. We use PCA to compute a statistical description of the shape model and to obtain the average vector of the positions m_{av} and the principal modes of variation $\boldsymbol{u_1,...,u_D}$. Considering a shape space as a weighted linear combination of the first $L \leq D$ eigenvectors $\boldsymbol{u_1,...,u_L}$, each element m that belongs to R^D in this shape space can be defined by a set of coefficients $\alpha_1,...,\alpha_L$ (Figure 1) as: $m = m_{av} + \sum_{i=1}^{L} \alpha_i \, sqrt(\lambda_i) \, \boldsymbol{u_i}$, where $\lambda_1,...,\lambda_L$ are the eigenvalues corresponding to each eigenvector of the PCA decomposition of a set of training shapes in R^D. We define as well a scalar mapping $\mu : A=[\alpha_{min},\alpha_{max}]^L \rightarrow R$ that can be any measure derived from the shapes in the PCA shape space, and can represent a clinically meaningful pathoanatomical criterion. Our goal is to find all instances in the shape space that meet a certain criterion dependent on a scalar measure μ. This problem is solved using level set segmentation on the shape space defined by the mapping μ. The level set segmentation allows for the representation of objects with complex topologies [4,5,6] and in our application it can be used to identify disconnected subsets of the shape space that meet the criterion. In order to segment the observed space μ we propose to minimize the following energy functional: $E(C) = a \int_w (\mu - M) \, dx + b \int_s |C'| \, ds$, where ω belongs to R^D, and $C(s)=\partial\omega/\partial s$ is a parameterised propagated surface embedded in the shape space. The first term represents the boundary force that attracts the evolving surface towards a predefined segmentation constraint $M = const$, while the second term regulates the smoothness of the surface. The zero level set computation is further optimised using automatic seed initialisation and narrow band level set evolution [7].

Results

We validate our method by an application to shape analysis of human femora. The results are obtained from a training set of 30 surface models extracted from CT data. These models represent complete left human femora. Correspondences across data sets were established with a spherical harmonic (SPHARM) based shape representation method. These correspondences are further optimized via a Minimum Description Length (MDL) optimization. The average shape was computed by simple averaging of corresponding landmarks across the data sets. The remaining variation was analyzed by PCA (Figure 2). We retain the first three principal components, which account for 89.22% of shape variability in the population. In our case, we use the range $-3 \leq \alpha_i \leq 3$ for every shape coefficient. This accounts for 99.7% of the shape variability encompassed in each principal component. We generate a scalar 3D map by computing the difference between the anteversion angles of the mean femur shape and the generated instance shape. We do not need to explicitly compute μ for every point in the shape space, but only in a narrow band around the zero level set, to reduce computational burden. Finally, the segmented area gives the set of shapes that have a similar range of anteversion angle (Figure 3). This

information can then be used by implant manufacturers to determine the best implant design to fit most of the population.

Conclusion

The method for optimisation in PCA shape space allows to find a partition of the shape distribution into regions that meet / do not meet a given criterion. Illustrative results have been shown for anatomical analysis of femora. Although the example has been elaborated for 3D maps (i.e. taking only 3 principal components), the method is applicable to maps of any dimension, determined by the number of principal components retained. To our knowledge, this is the first research into the problem of finding all instances in a shape distribution meeting a given criterion. The practical use of such a concept is of extreme importance in the study of the anatomical evidence of a pathology, or the morphologic features in implant positioning.

References

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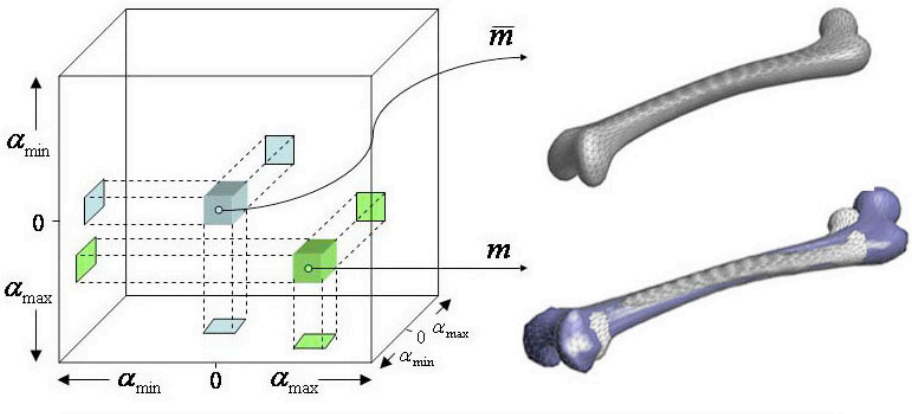

Fig.1. Shape space defined by the three first principal components. The center element m_{av} corresponds to the mean of the population. Each element m in this shape space is formed by a linear combination of the PCs $\mathbf{u_i}$: $m = m_{av} + \sum_{i=1}^{L} \alpha_i \, sqrt(\lambda_i) \, \mathbf{u_i}$.

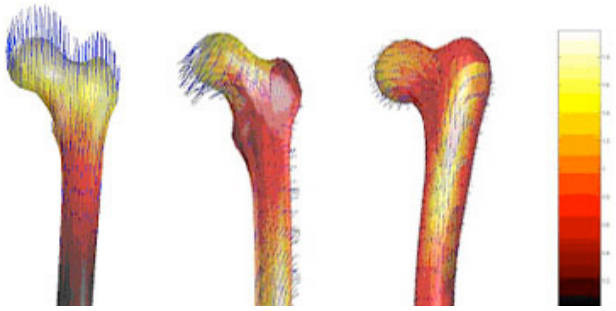

Fig.2. First three modes of variation for left femur. The lines represent the positive direction of of the principal component. The first mode describes the change of the femur length, second mode is related to the inclination of the femoral head and the third mode describes a deformation of the posterior part of the femoral head and a slight torsion and curvature of the central region.

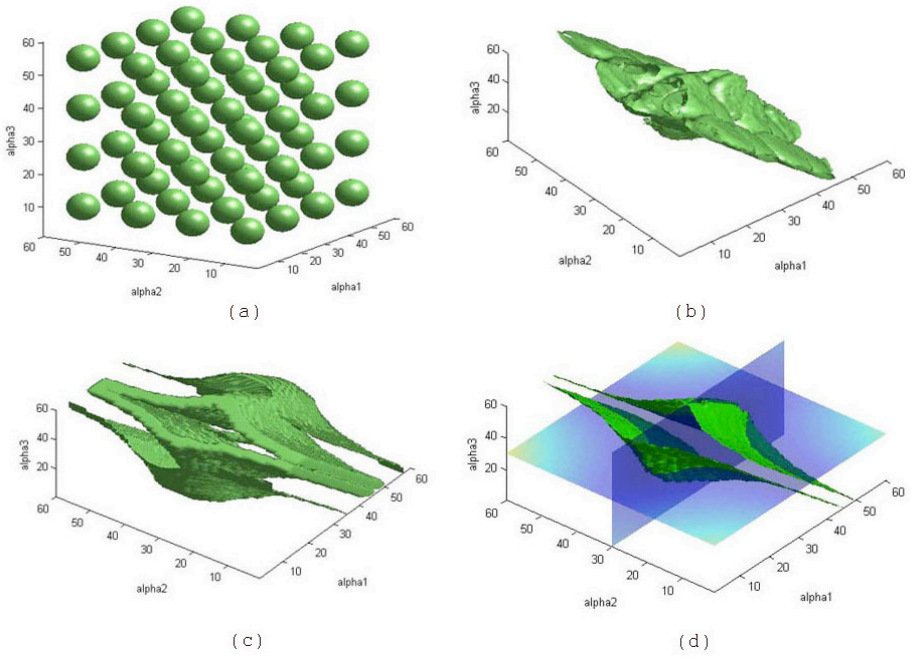

Fig.3. (a) Automatic 3D seed initialisation of the level set. (b) The zero level set during the evolution. (c) The narrow band around zero level set. (d) The final zero level set.