Influence of smoothing on voxel-based mesh accuracy in micro finite-element

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Abstract. The interest in automatic volume meshing for finite element analysis (FEA) has grown more since the appearance of microfocus CT (µCT), due to its high resolution, which allows for the assessment of mechanical behavior at a high precision. Nevertheless, the basic meshing approach of generating one hexahedron per voxel produces jagged edges. To prevent this effect, smoothing algorithms have been introduced to enhance the topology of the mesh. However, whether smoothing also improves the accuracy of voxel-based meshes in clinical applications is still under question. There is a tradeoff between smoothing and quality of elements in the mesh. Distorted elements may be produced by excessive smoothing and reduce accuracy of the mesh. In the present work, influence of smoothing on the accuracy of voxel-based meshes in micro FE was assessed. An accurate 3D model of a trabecular structure with known apparent mechanical properties was used as a reference model. Virtual CT scans of this reference model (with resolutions of 16µm, 32µm and 64µm) were then created and used to build voxel-based meshes of the microarchitecture. Effects of smoothing on the apparent mechanical properties of the voxel-based meshes as compared to the reference model were evaluated. Apparent Young's moduli of the smooth voxel-based mesh were significantly closer to those of the reference model for the 16 and 32µm resolutions. Improvements were not significant for the 64µm, due to loss of trabecular connectivity in the model. This study shows that smoothing offers a real benefit to voxel-based meshes used in micro FE. It might also broaden voxel-based meshing to other biomechanical domains where it was not used previously due to lack of accuracy. As an example, this work will be used in the framework of the European project ContraCancrum, which aims at providing a patient-specific simulation of tumour development in brain and lungs for oncologists. For this type of clinical application, such a fast, automatic, and accurate generation of the mesh is of great benefit.

Keywords: finite element, meshing, smoothing, validation, microfocus CT

1 Introduction

The high impact of osteoporosis on medical costs has made it a major branch of biomechanics. The development of new technology such as the micro CT and,

subsequently, of high resolution finite element studies has increased the understanding of this condition. The effects of new treatments on microscopic bone properties or prediction of fracture can be assessed with these methods for example.
However, the creation of the highly complex meshes used in the finite element analysis is a challenge. The voxel-based method is the easiest way to build such a mesh. In brief, it consists of associating a voxel representing bone with a cubic element in the mesh. Although these models are fast and automatic, the surfaces generated are composed of jagged edges. This is problematic for simulations, since stress concentrations may appear at sharp corners. Consequently the accuracy of these models can be low as compared to other methods [1].

One solution to improve the aspect of these models is to smooth its external surface. Smoothing of voxel-based FE meshes has been shown to improve accuracy in the simple case of a sphere [2]. However, the effect of this smoothing in real biomechanical applications, such as retrieval of bone structural properties has never been tested.

Therefore, the aim of this work was to assess the effect of smoothing on the accuracy of voxel-based mesh as compared to a reference model.

2 Methods

The influence of the degree of smoothing on the accuracy of finite element was evaluated by measuring the apparent Young's modulus of a trabecular structure. Results obtained with a reference model were compared to voxel-based models with different degrees of smoothing.

2.1 Creation of the reference model

A trabecular bone specimen from a human vertebra was scanned in the axial plane with a high resolution scanner (Scanco μCT40, Scanco Medical AG, Switzerland). The voxels had a dimension of 8μm in every direction. The CT scan obtained was segmented (Amira, Visage Imaging GmbH, Germany) and an accurate 3D model with 1,286,444 triangles was generated. The triangular surface mesh of the trabecular bone was then processed further in a CAD program, and fitted with a set of higher order mathematical surfaces e.g. NURBS (Non-Uniform Rational B-Splines). The smooth model of the trabecular structure obtained was considered as the reference geometry (figure 1).

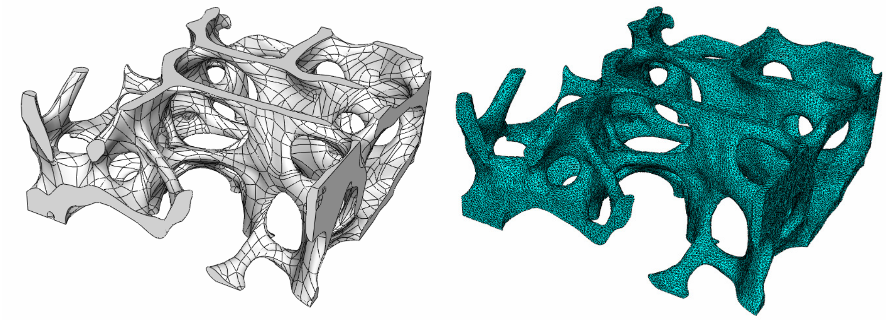

Fig. 1. (left) NURBS and (right) quadratic tetrahedral element mesh used as reference model for the study.

The NURBS model obtained was imported into a commercial finite element software (ABAQUS, Simulia, USA) and a volumetric finite element mesh with quadratic tetrahedral elements was generated. To assess the accuracy of this reference model, a convergence study with different mesh densities was performed. Total strain energy of the mesh under an axial loading was evaluated and compared. Meshes with a number of elements ranging from 28537 to 919780 were tested. Following this convergence study, a mesh with 559814 quadratic tetrahedral elements was chosen as the reference model.

2.2 Creation of the voxel-based mesh

In order to compare the reference model with voxel-based meshes, a virtual CT scan of the NURBS geometry was created using Amira. Resolution of the virtual CT scan was similar to the original one (8µm voxel size). The CT scan was resampled at voxel sizes of 16 µm, 32 µm and 64 µm, producing voxel-based meshes of 231071, 29169 and 3529 elements respectively. These resolutions are typically used for *in vivo* micro CT.

2.3 Smoothing

The outer surface of the mesh was extracted and smoothed according to the geometric signal processing approach presented in [3]. Let the column vector x be the vector of either the first, second or third coordinates of the vertices. The laplacian operator is defined on this graph signal by:

$$\Delta x_i = \sum_{j \in i^*} w_{ij} \left(x_j - x_i \right) \tag{1}$$

with i^* being the neighborhood of the vertex i and w_{ij} the weights of the operator. The laplacian operator can be written under a matrix form

$$\Delta x = -Kx \tag{2}$$

With $K = I - W$, $W = (w_{ij})$, and with its elements equal to zero if j is not a neighbor of i. The eigenvectors e^j of the matrix K define the natural vibration mode of the graph and form a basis of a n-dimensional space in which the signal x can be decomposed as:

$$x = \sum_{i=1}^{n} \hat{x}_j e_j \tag{3}$$

This formulation is equivalent to the Discrete Fourier Transform of the signal x. The smoothing of the surface is then performed by applying a low pass filter with transfer function $f(K)$:

$$x' = f(K)\,x = \sum_{i=1}^{n} f(k_i)\,x_i \cdot e_i \tag{4}$$

With $0 \le k_1 \le k_2 \le \cdots \le k_n \le 2$ being the eigenvalues of the matrix K. The window sync low-pass filtering transfer function is then approximated using Chebyshev polynomials:

$$T_n(w) = \begin{cases} 1 & n = 0 \\ w & n = 1 \\ 2wT_{n-1}(w) - T_{n-2}(w) & n > 1 \end{cases} \tag{5}$$

The advantages of using this approximation are that the terms of the polynomial are orthogonal, it needs small storage capacities (i.e. three-term storage), it is numerically stable, and it can be defined for volume preservation purposes.

Smoothing was performed up to a pass-band frequency of k=0.03. Further smoothing induced negative volume elements.

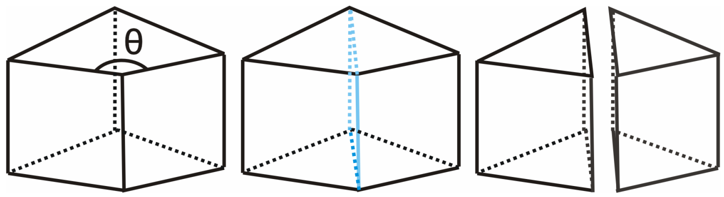

Fig. 2. Correction of elements featuring large angles (here represented by θ). The elements are divided into two prism elements along the plane that passes through the "large angle" edge.

2.4 Prism division

Extensive smoothing creates distorted elements with large or very small dihedral angles on the surface of the mesh. To improve the quality of the mesh, hexahedral elements bearing a large angle between faces were divided into prism elements (figure 2). If the angle was superior to a certain value, then the edge at the intersection of the two faces was used for the division. The element was divided by the "virtual" plane that joined this edge with its opposite in the element. Improvements were measured by counting the number of distorted elements having interior angles between isoparametric lines less than 45 ° and greater than 130 °. Since it produced the best improvements (tradeoff between increase in the number of elements and improvement in general mesh quality), a threshold angle of 130 ° was chosen for the splitting.

2.4 Finite element study

Once the smoothing was performed on the voxel-based mesh, the apparent Young's modulus was calculated in the three directions X, Y, Z, with Z the axis perpendicular to the axial plane of the vertebra. A 1% strain was applied on the top of the structure while the bottom was constrained in the direction of displacement. No other faces were constrained. The apparent Young's modulus *E* was calculated according to the formula:

$$E = \frac{Fl}{\Delta u A} \quad (6)$$

with Δu the applied displacement at the top of the structure, F the reaction force at the moving nodes, l and A respectively the length and the area as measured by the external dimensions of the specimen. For all models, element material properties were assumed homogeneous and isotropic with a Young's modulus of 10GPa and a Poisson ratio of 0.3.

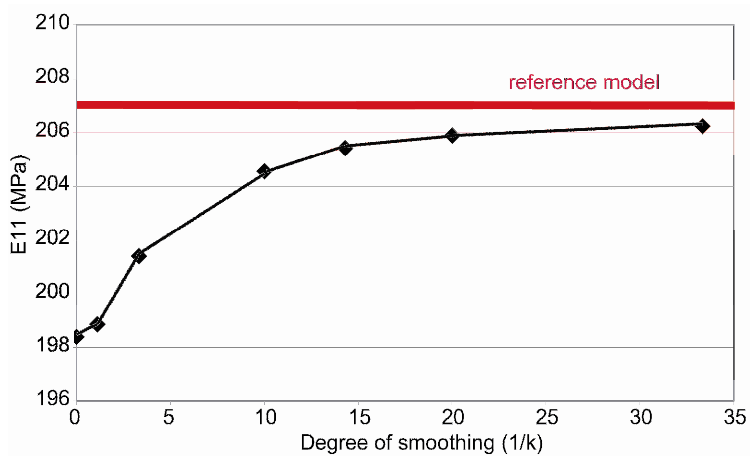

Fig. 3. Evolution of the apparent Young's modulus in the X direction for the 16μm model with different degrees of smoothing.

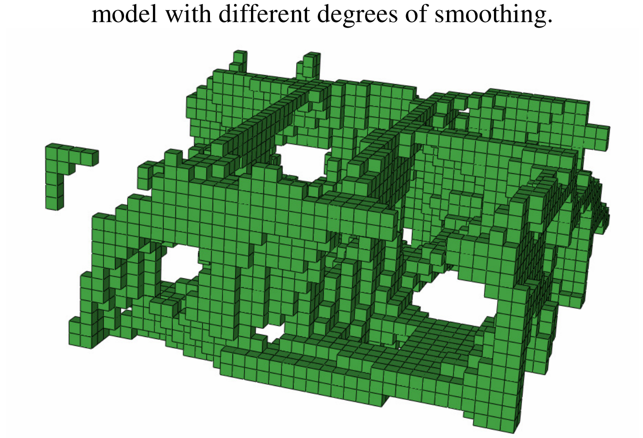

Fig. 4. Loss of trabecular connection for the 64μm model. This phenomenon explains the large error found for the apparent Young's modulus at this resolution.

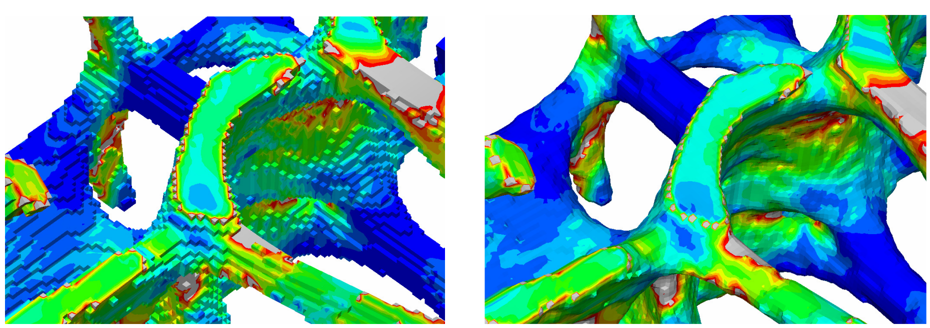

Fig. 5. Details of the Mises stress in the trabecular structure generated (left) without smoothing and (right) with smoothing.

3 Results

Increasing the degree of smoothing improves the mechanical properties of the voxel-based mesh as compared to the reference model (figure 3). Smoothing significantly improved the apparent Young's modulus of the voxel-based meshes for the 16 and 32µm resolutions (table 1). Improvements were not significant for the 64µm resolution. This is due to loss of trabecular connection in the model (figure 4). Visually, reduction of stress concentration can be observed on the edges of the model (figure 5). The volume of the model was preserved during the smoothing and changed only by 0.02% (for k=0.03 and the 16µm model). In average, creation of the mesh and smoothing took 3 minutes for the 16 µm model on a regular 2.4GHz processor with 2.00GB of RAM.

As expected [4], the Young's modulus in the Z direction (≈572MPa) was found to be superior to the Young's modulus in the directions parallel to the axial plane of the vertebra (≈260 and 257MPa respectively).

Prism division significantly decreased the number of distorted elements (table 2). However, results in terms of accuracy were not significantly changed with this improvement with an average difference of 0.19%±0.17 as compared to the model without prism division. For k=0.03, the number of elements in the 16µm model was increased by 20% when prism division was used.

Table 1. Error (in % of reference value) for apparent Young's modulus between reference model and voxel-based meshes without smoothing and with maximum smoothing. Smoothing was stopped when negative volume elements were created.

	16µm			**32µm**			**64µm**		
	E11	**E22**	**E33**	**E11**	**E22**	**E33**	**E11**	**E22**	**E33**
No smoothing	4.15	3.41	5.74	8.02	15.63	10.71	12.10	33.99	23.19
k=0.03	0.36	0.97	2.17	1.32	8.67	5.19	12.25	32.37	21.44

Table 2. Comparison of the number of distorted elements generated by smoothing with and without prism division. Elements were considered distorted when the interior angles between isoparametric lines was inferior to 45 ° and superior to 130 °.

	Number of distorted elements without prism division	Number of distorted elements with prism division
k=0.1	2449	1
k=0.07	8173	1
k=0.05	13569	0
k=0.03	18901	1

4 Discussion

Voxel-based meshing is the method of choice for micro FE due to its speed and low complexity. However, it does not represent smooth anatomy with accuracy [1]. Smoothing algorithms have been proposed to improve these meshes [2] but their influence on real anatomical models has never been assessed. Therefore, in this study, the effect of smoothing on the mechanical properties of a voxel-based mesh of bone was tested.

The apparent Young's modulus was significantly improved by smoothing and converged to the reference value. This is in accordance with previous studies on simple models that showed that the accuracy of the finite element meshes was improved by smoothing [1], [2]. With smoothing, voxel-based meshes can reach an accuracy equivalent to more complex tetrahedron models. Results achieved with the smooth 32 µm model were comparable to those obtained with the non smooth 16 µm. This would suggest that smoothing allows one to reduce the resolution of images and consequently to significantly reduce the number of elements in the mesh. However, the effect of smoothing is relevant only for image resolutions where the connection of the trabeculae is kept intact. In our case, loss of connection happened at a resolution of 64 µm and explains the large errors found for the Y and Z directions [5]. In these cases, smoothing does not correct errors due to unconnected areas. The high dependence on image resolution is one clear limitation of the voxel-based meshes [6]. One solution that was proposed in the literature was to change the threshold used for the segmentation of the CT scan and to compensate for the loss of connected areas by thickening the remaining structure [5]. This was not possible in the present study since virtual CT scans were used. The combination of smoothing and change in the segmentation threshold should be assessed in a future study.

Splitting the distorted elements into prisms significantly improved the quality of the mesh at limited computational costs. Therefore, it was possible to smooth the mesh without concern for the quality of the elements. In existing voxel mesh software (Simpleware, Simpleware Ltd, UK) every element on the surface is unconditionally split into tetrahedra. In our case, elements which are to be split or corrected are discriminated. Hence, the augmentation of elements after division is significantly lower since it only represents a small proportion of elements in the mesh. Moreover, division into prisms generates fewer elements than division into tetrahedra. The fact that improving element quality did not have an influence on result of the analysis may be due to the global aspect of the apparent Young's modulus. Local analyses such as surface strain measurements would clearly be more influenced by element quality [6].

Smoothing was limited to a certain degree, due to generation of negative volume elements. Regularization algorithms that would move interior nodes in order to improve element quality and take smoothing further are currently under investigation. One simple solution to this could also be to remove elements with negative volume (as long as their number is small). However, the question arises: What effect will extensive smoothing have on the accuracy of the model, since it might induce the loss of important geometrical features? In that case, a limit would have to be defined for the smoothing. Future work will also consist of comparing mechanical properties of the mesh with more virtual models but also with real experimental data.

This study shows that smoothing has a real application in voxel-based meshes used in micro FE. Smooth voxel-based meshes could be used in other applications such as statistical shape models of full bones, for example, where fast and automatic generation of an accurate mesh is of great benefit. Use of voxel-based mesh in these situations has been assessed but did not lead to significant results [6]. Addition of smoothing would greatly improve the accuracy of these models and allow their use in various domains. For this reason, this algorithm will be used in the european project ContraCancrum which aims to develop an oncosimulator for clinicians. Fast, automatic, and robust algorithms such as presented in this study are particularly adapted for this type of clinical applications.

Acknowledgements

Funding by the European Union in the framework of the ContraCancrum project (FP7 -- IST-223979) is gratefully acknowledged.

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